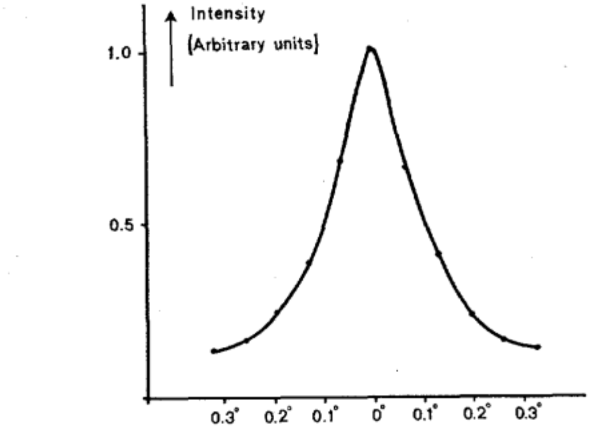

Fig. 2. Second-harmonic power as a function of the external angle of deviation from the phase-matching conditions.

the x axis, we find for the second harmonic power, using MKS units,

$$P(2\omega) = 2\left(\frac{\mu}{\epsilon}\right)^{3/2} \omega^2 L^2 \, d_{eff}{}^2 \, \frac{P^2(\omega)}{\pi \omega_0{}^2} \, G(t, q) \qquad (1)$$

where

- $P(\omega)$ fundamental power,
- ω fundamental frequency,
- d_{eff} $|d_{31}| \sin \Theta_m + |d_{22}| \cos \Theta_m$,
- ω_o beam waist radius,
- L length of the crystal,
- $G(t, q)$ correction due to birefringence and absorption as defined by Boyd *et al.* [9].

Substituting values for our experimental conditions gives

$$G(t, q) = G(2.84, 0.05) = 0.65. \qquad (2)$$

Substituting the numerical values we find

$$P(2\omega) = 6.4 \times 10^{37} \, d_{eff}{}^2 P^2(\omega). \qquad (3)$$

From the observed second-harmonic power we then deduce

$$d_{eff} = (7.9 \pm 2.0) \times 10^{-23} \qquad (\mathrm{MKS}), \qquad (4)$$

which is a smaller value than that obtained from a mixing experiment in the 1-μ wavelength region [6].

In Fig. 2 we have plotted the second-harmonic power as a function of the external angle of deviation from the phase-matching condition. From this we deduced the half-width to be 0.22°. According to calculations using the dispersion of the refractive indices [8] it should be a factor of 2 larger.

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Parametric Oscillator Tuning Curve from Observations of Total Parametric Fluorescence

J. E. PEARSON, U. GANIEL, AND A. YARIV

***Abstract*—Measurements of total emitted parametric fluorescence power are presented and used to fix one point on the predicted tuning curve of a parametric oscillator. The method is particularly useful for predicting the tuning curve of infrared pumped parametric oscillators. Experimental results, which verify the usefulness of the technique in a 1.06-μ-pumped oscillator, are presented.**

Introduction

Optical parametric fluorescence has been studied extensively both theoretically [1]–[4] and experimentally [3], [5]–[8]. One application of parametric fluorescence observations is in obtaining the tuning curve for an optical parametric oscillator, but because of low fluorescence power and small monochromator bandwidth, measurements to date have been made only in the UV to near IR regions where at least one of the generated wavelengths falls in a range where photomultipliers can be used. We present here a technique for predicting the tuning curve of a parametric oscillator using known index of refraction data and a measurement of total emitted fluorescence power. This method is especially useful for infrared-pumped oscillators where high sensitivity detectors are not available and the fluorescence power is greatly diminished. The results reported here were obtained using a Q-switched laser, but the method is equally applicable to CW pump sources.

Theory

From the plane-wave theory of Byer and Harris [3], the total signal-plus-idler parametric fluorescence power incident on a detector with acceptance angle θ is

$$P_T = L^2 K P_3 \int_{-\infty}^{\infty} \int_0^{\theta} \frac{\sin^2 (\Delta k L/2)}{(\Delta k L/2)^2} \, \phi \, d\phi \, d\omega \qquad (1)$$

where L is the nonlinear crystal length, P_3 is the pump power in watts, and $\Delta k = |\boldsymbol{k}_3 - \boldsymbol{k}_1 - \boldsymbol{k}_2|$ is the wavevector mismatch. The constant K is given in [3]. For small angles Δk can be expanded in a Taylor series about the collinear phase-matched frequencies. The result to second order is

$$\Delta k = -b_o \omega - b_1 \omega^2 + g\phi^2 \qquad (2)$$

$$g = \frac{k_{10} k_3}{2k_{20}}, \qquad \omega \equiv \omega_1 - \omega_{10} = \omega_{20} - \omega_2 \qquad (3)$$

$$b_o = \left.\frac{\partial k_1}{\partial \omega_1}\right|_{\omega_{10}} - \left.\frac{\partial k_2}{\partial \omega_2}\right|_{\omega_{20}} \qquad (4)$$

$$b_1 = \frac{1}{2}\left[\left.\frac{\partial^2 k_1}{\partial \omega_1{}^2}\right|_{\omega_{10}} + \left.\frac{\partial^2 k_2}{\partial \omega_2{}^2}\right|_{\omega_{20}}\right] \qquad (5)$$

where ω_{10} and ω_{20} are the collinear signal and idler frequencies such that $\omega_3 = \omega_{10} + \omega_{20}$, ω is the deviation from the collinear frequencies, and ϕ is the angle between $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$.

A careful examination of (1) and (2) shows that the total power P_T peaks as the fluorescence is tuned toward collinear degeneracy. The location of the peak fluorescence power when P_T is plotted as a function of the tuning parameter (the crystal temperature, for example) will vary depending on the

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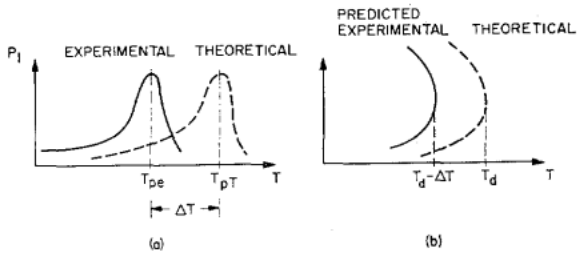

Fig. 1. Procedure to obtain predicted parametric oscillator tuning curve from total fluorescence power measurement. (a) Experimental and theoretical plots of total fluorescence power versus temperature define the shift ΔT between the peaks. (b) Theoretical phase-matched wavelength versus temperature curve is shifted by ΔT to obtain the predicted oscillator tuning curve.

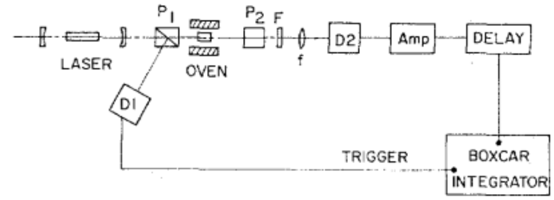

Fig. 2. Schematic of experimental setup for measuring fluorescence. Pumping laser—1.06-μ repetitively Q-switched Nd:YAG. $P1$, $P2$—crossed polarizers. F—three 1.06 reject, 1.3–3.0-μ pass filters. $D1$, $D2$—high-speed detectors. f—short focal length lens. The $LiNbO_3$ crystal is held in a temperature-controlled oven.

acceptance angle θ. Physically, the reason the total power peaks is that the power per unit bandwidth is approximately constant [3] while the total bandwidth in the solid angle $\pi\theta^2$ first increases and then begins to decrease as collinear degeneracy is approached. In fact, the peak fluorescence power can be identified with the point at which the degenerate frequencies begin to fall within the detector acceptance angle [9]. The detailed behavior of how the fluorescence power peaks is dependent on the sign of the coefficient b_o. An example of the peaking of the total fluorescence power is shown in Fig. 3 for a 1.06-μ pump and $LiNbO_3$ as the nonlinear crystal (b_o is negative for this case).

If the indices of refraction are known for a particular crystal, the tuning curve of a parametric oscillator using that crystal can be calculated. The indices of refraction of crystals such as $LiNbO_3$, however, are known to vary with composition [10], [11] so that the exact tuning curve of a particular crystal may not be described by the published values of the refractive indices. If, however, the shape of the tuning curve is unchanged by different crystal composition, the peaking of total fluorescence power can be used to fix one point on the tuning curve and thus allow the entire tuning curve to be predicted. The procedure is illustrated in Fig. 1 for the case of temperature tuning. The predicted oscillator tuning curve is shifted along the temperature axis by the amount ΔT, which is the temperature shift required to align the peaks of the theoretical and experimental fluorescence curves. The close agreement between theory and experiment in Fig. 3 gives support to the assumption that the shape of the tuning curve is unaffected by composition differences between crystals.

Experimental

In order to verify the validity of the above technique, the total fluorescence power emitted from a $LiNbO_3$ crystal was measured as a function of temperature. The pump was a Q-switched Nd:YAG laser operating at 1.064 μ. The experimental setup is shown in Fig. 2 and the results of the measurements are shown in Fig. 3. Note that the peak fluorescence power *does not* occur at collinear degeneracy as suggested in previous observations of total emitted power [12]. The theoretical curve in Fig. 3 is computed from the data in [13] and has been shifted by $-29.5°C$ along the temperature axis to align the theoretical with the peak experimental maximum.

Using the same crystal as in the fluorescence experiment, a 1.06-μ-pumped $LiNbO_3$ parametric oscillator has been constructed. The oscillator mirrors were coated directly on the ends of the plane-parallel crystal faces and the crystal, held in a temperature-controlled oven, was placed internal to the laser cavity. The tuning curve of this oscillator for three sets of mirrors is shown in Fig. 4 along with the theoretical

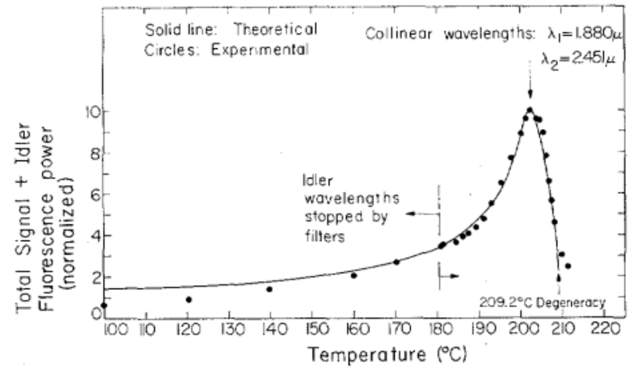

Fig. 3. Theoretical and experimental plots of total fluorescence power versus temperature in $LiNbO_3$ with $\lambda_3 = 1.064$ μ, a phase-match angle of 50°, and a maximum detector acceptance angle of 1.00°. Each curve has been normalized to its respective maximum and the theoretical curve computed using index data in [13] has been shifted by $-29.5°C$ to align the peaks of the two curves. For temperatures below that indicated, only the signal power is included in the theoretical curve because of the reduced transmission of the filters used in the experiment.

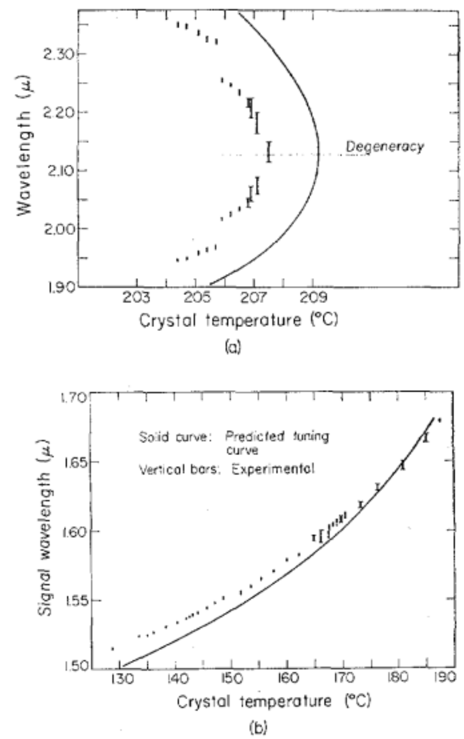

Fig. 4. Comparison of experimental parametric oscillator tuning curve and tuning curve predicted using fluorescence data and index data in [13]. Vertical bars show total oscillating bandwidth at the indicated temperature. The theoretical curve has been shifted by $-29.5°C$. (a) Oscillator mirrors highly reflecting around 2.12 μ. (b) Mirrors highly reflecting around 1.6 μ and 3.2 μ. Data from two different sets of mirrors are included.

tuning curve shifted by $-29.5°C$. The agreement between theory and experiment is reasonably good, with the degenerate point being predicted to within 1.7°C. The small deviations between theory and experiment can be attributed to slight differences in alignment and to differences between the actual and theoretical indices of refraction at high temperatures and long idler wavelengths.

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Controlled Bistable Operation of an FM Mode-Locked Nd:YAG Laser

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***Abstract*—It is well known that there are two possible phase positions for the pulse in a mode-locked laser with an internal phase modulator. By using a modulator cut in the form of a Brewster angle prism, it becomes possible to operate an Nd:YAG laser stably in either position at will. The angular beam deflection is slightly different for the two modes and, hence, by mirror adjustment one mode or the other can be selected. A simple analysis is presented to estimate the magnitude of this effect in rough agreement with experimental observations.**

In a mode-locked laser with an internal-phase FM modulator, there are two possible phase positions for the pulses, which are separated by 180° in phase with respect to the modulator driving signal. The two solutions come about because a short pulse can pass through the modulator at either extreme of the sinusoidal phase variation and thereby not experience any Doppler shift. One usually finds that the output of a FM mode-locked laser jumps back and forth randomly between these two solutions. This can be extremely troublesome for some applications such as pulse-code modulation communication.

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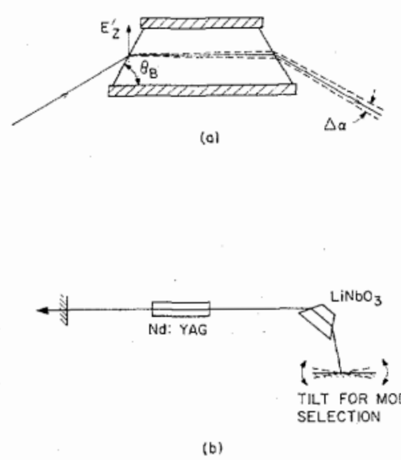

Fig. 1. (a) Brewster angle modulator showing different deflection for two modes of FM mode-locked laser. (b) Nd:YAG laser with modulator showing mode selection by mirror adjustment.

This effect has been studied in detail by Hong and Whinnery [1] for the He–Ne laser. They found that under certain conditions the laser will operate stably in one mode only. However, in the Nd:YAG laser, this behavior is not usually observed. A theoretical analysis [2], [3] shows that the two solutions are essentially degenerate with only a very small difference in exact modulation frequency. In practice it is found that the 180° phase ambiguity and the resulting unstable behavior remain even if the modulator is detuned ie., [2b], [3]. In this letter a solution to this problem applicable in particular to the Nd:YAG laser, and in general to any FM mode-locked laser, is described.

To select a single-phase position in the FM laser, it is necessary to introduce some additional amplitude modulation in the cavity with a minimum-loss condition occurring only once during every cycle of the phase modulation. This can be done with the phase modulator alone by using a modulator crystal cut as a Brewster prism with an electrode structure as shown in Fig. 1(a). In this case the sinusoidal modulation of the index of refraction will cause both phase modulation and also some small beam deflection. The beam deflection will be at its minimum or maximum for the two preferred pulse positions, since these positions occur either at the extreme positive or negative value of the modulating signal. When this modulator is placed inside an optical cavity, the small periodic beam deflection causes a periodic variation in cavity alignment, which can then lead to the desired additional amplitude modulation. It becomes possible to select one mode or the other by a small tilt of one mirror, as shown in Fig. 1(b), so that the cavity mirror alignment is perfect at one pulse position and slightly misaligned at the other.

We can estimate the magnitude of this effect. In particular we will consider $LiNbO_3$ with the voltage applied along the z axis, a common configuration for Nd:YAG lasers. With a voltage V_0 applied to a crystal d cm thick and l cm long, the single-pass phase retardation is $\delta_m = \pi r_{33} n_e^3 V_0/\lambda_0 (l/d)$. With an electric field E'_z in the z direction on the crystal face, the change in index is $\Delta n_z = \frac{1}{2} r_{33} n_e^3 E'_z$. For a crystal with Brewster angle faces, the deflection $\Delta\alpha$ is $\Delta\alpha = 2\Delta n$. For a crystal with a high dielectric constant, the electric field E'_z as $E'_z \cong (V_0/d) \sin^2 \theta_B$ can be estimated. Hence, we can obtain the ratio of the deflection over the single-pass retardation, $\Delta\alpha/\delta_m = \lambda_0 \sin^2 \theta_B/\pi l$. For an $LiNbO_3$ crystal 1 cm long at 1.06 μ, $\Delta\alpha/\delta_m \cong 0.28 \times 10^{-4}$ rad is obtained. As expected, the effect is very small.

In an experimental setup for an Nd:YAG laser, with 5 W of modulating power in the modulator at 344 MHz, 1.2 rad